

REPUBLIC OF RWANDA



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ADVANCED LEVEL MATHEMATICS CURRICULUM FOR SCIENCE COMBINATIONS

Kigali, April 2010

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I. INTRODUCTION

After completing the curriculum of Mathematics for ordinary level, the curriculum of Mathematics for advanced level comes for capacity building of students in science combinations: MCB (Mathematics-Chemistry-Biology), MPG (Mathematics-Physics-Geography), MPC (Mathematics-Physics-Computer Sciences), MEG (Mathematics-Economics-Geography), MCE (Mathematics-Computer Science-Economics) and PCM (Physics-Chemistry-Mathematics). This curriculum is the revision of the curriculum of Mathematics for advanced level, edition 1999 and deals specifically with logic, trigonometry, analysis, algebra, geometry, statistics and probability. The chapters are developed in a logical progressive sequence enabling the learner to have a good comprehension of the subject matter.

This Mathematics curriculum is prepared in a format which helps teachers to teach a particular topic effectively. The structure of each chapter is presented in three columns:

- Specific objectives;
- Contents
- Suggested Teaching and Learning Activities;

At the end of detailed content of each grade, there is a proposal of lesson distribution.

To avoid the areas of Mathematics to be continually seen as separate and learners acquiring concepts and skills in isolation, Mathematics is linked to everyday life and experiences in and out of school. Learners will have the opportunity to apply Mathematics in different contexts, and see the relevance of Mathematics in daily life.

This curriculum also helps learners to use ICT tools to support the mastery and achievement of the desired learning outcomes. Technology used in the teaching and learning of Mathematics, for example calculators, are to be regarded as tools to enhance the teaching and learning process and not to replace teachers.

II. GENERAL OBJECTIVES BY THE END OF A' LEVEL

After the completion of advanced level secondary education, science combinations (PCM, MCB, MPC, MPG, MCE, and MEG) learner should be able to:

1. Develop clear, logical, creative and coherent thinking;
2. Master basic mathematical concepts and to use them correctly in daily life problem solving;
3. Express clearly, comprehensibly, correctly and precisely in verbal and/or in written form all the reasons and calculations leading to the required result whenever finding a solution to any given exercise;
4. Master the presented mathematical models and to identify their applications in the learner's environment;
5. Arouse learner's mathematical interest and research curiosity in theories and their applications;
6. Use the acquired mathematical concepts and skills to follow easily higher studies (Colleges, Higher Institutions and Universities);
7. Use acquired mathematical skills to respect human rights;
8. Use acquired mathematical skills to develop work spirit, team work, self-confidence and time management without supervision;
9. Use ICT tools to explore Mathematics (examples: calculators, computers, mathematical software,...).

III. LEARNER’S SKILLS TO BE IMPROVED

While teaching, the teacher should make sure that the skills listed below are developed for each topic in each grade through teaching and learning activities.

Skills	Main learning activities
1. ICT skills as tools for learning	<ul style="list-style-type: none"> • Using computers and projectors in presenting individual or group activities • Using calculators in operations
2. Communication skills	<ul style="list-style-type: none"> • Discussion in group, oral and writing presentations of findings (results), • Using mathematical language in presenting word problems
3. Individual skills	<ul style="list-style-type: none"> • Organize individual research (in the library) in a given time • Organize individual activities (exercises, homework, test,...) in a given time
4. Critical and logical thinking skills	<ul style="list-style-type: none"> • Using formulae and theorems to solve problems • Relating the solution of a problem to the real world
5. Critical and interpretation skills	<ul style="list-style-type: none"> • Collecting data, analyzing data, synthesizing data, interpreting data and presenting data by using tables, charts, diagrams, graphs,...
6. Group learning skills and Practical skills	<ul style="list-style-type: none"> • Organization of group activities • Following instructions in solving problems
7. Creative and innovation skills	<ul style="list-style-type: none"> • Activities of demonstration and generalization
8. Higher cognitive skills	<ul style="list-style-type: none"> • Various activities requiring high order thinking
9. Social skills	<ul style="list-style-type: none"> • Working in groups
10. Discernment/evaluation of information skills	<ul style="list-style-type: none"> • Self evaluation activities (exercises with final answers)
11. Problem solving skills	<ul style="list-style-type: none"> • Activities related to daily events (economic growth, productivity, ...)
12. Motivation and self confidence skills	<ul style="list-style-type: none"> • Activities related to the use of Mathematics in real life

IV. METHODOLOGICAL NOTES

The use of teaching resources is crucial in guiding learners to develop mathematical ideas.

Teachers should use real or concrete materials to help learners gain experience, construct abstract ideas, make inventions, build self confidence, encourage independence and inculcate the spirit of cooperation.

In order to assist learners in having positive attitudes and personalities towards Mathematics, confidence and thinking systematically have to be involved into the teaching and learning process. Good moral values can be cultivated through suitable contexts. Learning in groups should be emphasized to help learners to develop social skills, encourage cooperation and build self confidence. The element of patriotism should also be developed through the teaching and learning process in the classroom using examples.

Various teaching strategies and approaches such as direct instruction, discovery learning, investigation, guided discovery or other methods must be incorporated. Among the approaches that can be given consideration include the following:

- Learner-centered learning;
- Different learning abilities and styles of learners (individualization);
- Usage of relevant, suitable and effective teaching materials;
- Formative evaluation to determine the effectiveness of teaching and learning process.

The choice of a suitable approach will stimulate the teaching and learning environment inside or outside the classroom. The considered suitable approaches include the following:

- Cooperative learning;
- Contextual learning;
- Mastery learning;
- Constructivism.

In this curriculum, suggested various exercises in all chapters may be done in groups or individually.

In implementation of this curriculum, some activities to be done should be related to the main courses (core subjects) of each combination in order to establish the relationship between Mathematics and other subjects.

Examples:

- ♦ In MCE (Mathematics, Computer Sciences and Economics) some given activities in statistics, in functions, in sequences,... should be related to Economics.
- ♦ In MPG, (Mathematics, Physics and Geography) some given activities in statistics, in logic, in functions,... should be related to Physics or Geography.

V. LIST OF TEACHING AIDS

1. Geometric instruments:
 - Ruler,
 - Compass,
 - Protractor,
 - T-square
2. Graph paper, Flip chart, wall Charts, ...
3. Calculators , computers and Interactive multimedia content
4. Cubic die

VI. EVALUATION APPROACH

Evaluation or assessment has to be planned and carried out as a part of the classroom activities. Different methods of assessment can be conducted. These may be in the form of assignments, oral questioning and answering, observations and interviews. Based on the given responses, teachers can rectify learners' misconceptions and weaknesses and also improve his/her own teaching skills. Teachers can then take subsequent effective measures in conducting remedial and enrichment activities in upgrading learners' performances.

VII. PROGRAMS

PROGRAM FOR SENIOR 4

GENERAL OBJECTIVES BY THE END OF SENIOR FOUR

At the end of senior 4 the learner should be able to:

1. Utilize correctly vocabularies and symbols of mathematical logic in different demonstration methods;
2. Solve algebraically and graphically given problems by applying properties of operations in the field of real numbers;
3. Use algebra and vectors in solving problems related to plane geometry;
4. Use the trigonometric concepts and formulas in solving problem related to trigonometry;
5. Determine and interpret the dispersion parameters of statistical series in one variable;
6. Carry out different operations on matrices of order 2 and order 3

CHAPTER I: LOGIC

❖ **Duration: 14 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To utilize correctly vocabularies and symbols of mathematical logic in different demonstration methods.

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> - To use correctly propositions to determine whether a given sentence is a mathematical statement using definition - To determine whether a given statement is true or false - To write the negation of a given proposition 	<p>1. Logic</p> <p>1. 1.Generalities:</p> <ul style="list-style-type: none"> - Propositions - Truth table of a proposition - Negation of a proposition 	<ul style="list-style-type: none"> - A statement is also called a Proposition - The teacher should lead learners to determine whether a given sentence is a mathematical statement through different examples - The teacher should give learners various exercises to construct the truth table of propositions - Through various exercises, use the word “not” or “no” and their symbols to change a true statement into a false statement and vice versa

<ul style="list-style-type: none"> – To form a compound statement from two simple statements by using the conjunction “and” – To determine the truth value of the compound statement – To form a compound statement from two simple statements by using the disjunction “or” – To determine the truth value of the compound statement – To construct mathematical statements by using implication sign (\Rightarrow). – To determine the converse of a given implication and determine whether it is true or false. – To apply correctly Morgan’s law – To determine whether a statement is a tautology or not. – To determine if a statement is true or false using correctly quantifiers (\forall) and (\exists) 	<p>1.2.Logic of propositions :</p> <ul style="list-style-type: none"> – Logical connectives and their properties: <ul style="list-style-type: none"> ▪ conjunction ▪ disjunction ▪ implication ▪ equivalence – Truth tables of propositions including connectives – Morgan’s Laws – Tautology – Propositional forms in one or several variables – Referential of a propositional form – Universal quantifier (\forall) – Existential quantifier (\exists) – Negation of propositions including quantifiers. 	<ul style="list-style-type: none"> – Help learners to discuss in groups using logical connectives “or”; “and” – Lead learners to differentiate the use of “If p, then q” and “p if and only if q” through various exercises – Make sure that the students are able to identify whether a proposition is a tautology or a contradiction through different exercises – Lead learners to write statements involving universal and existential quantifiers using mathematical symbols through various exercises
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CHAPTER II: ALGEBRA

❖ **Duration: 84 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To solve algebraically and graphically given problems by applying properties of operations in the field of real numbers.

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To carry out correctly the operations in sets of numbers. 	<p>2. Algebra</p> <p>2.1. Sets of numbers and their structures</p> <p>2.1.1. Revision on sets of numbers : N:Natural numbers , Z: integers, Q :rational numbers, R : real numbers :</p>	<ul style="list-style-type: none"> – The teacher should emphasize on inclusion of sets of numbers $\square \subset \square \subset \square \subset \square$ and give concrete examples to explain the insufficiency of

	Operations on these sets of numbers.	each set – All activities to be done should verify properties in operations: closed, commutative, associative and distributive properties,...
- To factorize polynomials by using different methods - To simplify algebraic expressions and function	2.1.2. Polynomials : factorization, expansion of operations on fractional algebraic expressions and functions	– The teacher should prepare different and various exercises involving different methods of factorization such as: common factor method, grouping method, Horner’s method and use of identities. – Before adding or subtracting algebraic fractions ensure that the denominators are the same using Lowest Common Multiple (LCM)
- To determine if a set with internal composition law is a GROUP, - To determine if a set with two composition laws is RING, or a FIELD - To determine if the set of real numbers with two composition laws is a vector space	2.1.3. Structures: Groups, Rings and Fields - Internal composition law, group, ring and field - External composition law and structure of a real vector space	– Organize group activities to prove if a given set with internal/external composition laws is a group, a ring or a field using their properties
- To apply properties of the absolute value. – To simplify algebraic expression using properties of indices and radicals.	2.1.4. Set of Real numbers : -absolute value and its properties: absolute value of a product, of a quotient, of a sum and of a difference – Indices or powers, and radicals (surds) : ▪ integer indices or integer powers and square roots ▪ the n^{th} root of a positive real number and its properties ▪ fractional indices and their properties	– The teacher should prepare various exercises on absolute value

<ul style="list-style-type: none"> - To solve system of equations in three unknowns - To represent graphically a solution of inequations and simultaneous inequations in two unknowns. - To solve problems involving a system of two or three equations 	<p>cramer's method</p> <ul style="list-style-type: none"> - Solution of system of equations involving three unknowns using linear combinations and substitution method - Solutions of system of homogeneous equations - Graphical solution of inequations and simultaneous inequations in two unknowns. - Word problems involving three and two unknowns 	<ul style="list-style-type: none"> - Give various exercises to solve systems of equations by using the acquired methods.
<ul style="list-style-type: none"> - To identify the quadratic equations with one unknown - To factorize a quadratic expression - To solve the quadratic equations - To determine the Sum and product of roots of quadratic equations - To study the Sign of $ax^2 + bx + c$ - To solve the quadratic inequations - To solve problems leading to quadratic equations 	<p>2.2.2. Quadratic equations and inequations:</p> <ul style="list-style-type: none"> - Definition of a quadratic equation - Factorization of $ax^2 + bx + c$ - Solution of quadratic equations $ax^2 + bx + c = 0$, - Sum and product of roots of quadratic equations - Sign of $ax^2 + bx + c$ - Solution of quadratic inequations - Examples of equations reducing to quadratic equations: biquadratic equations, reciprocal equations and simple irrational equations - Problems leading to quadratic equations 	<ul style="list-style-type: none"> - lead learners to use the expression $ax^2 + bx + c = a \left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right) \right]$ for determining the solution of quadratic equation - The teacher should give various exercises involving the use of sum and product of roots of quadratic equations - Emphasize the role of sign table in solving quadratic inequations
<ul style="list-style-type: none"> - To sketch the quadratic function graph 	<p>2.2.3. Quadratic functions</p> <ul style="list-style-type: none"> - Definition - Graphical representation of quadratic function 	<ul style="list-style-type: none"> - To plot, teacher should help learners to determine the concavity by considering the sign of a, vertex, axis of symmetry and intersection with axes

CHAPTER III: ALGEBRA AND PLANE GEOMETRY

❖ **Duration: 35 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:
To use algebra and vectors in solving problems related to plane geometry

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To determine the sum of vectors and the product of a vector by a real number. – To verify that $(\mathbb{R}, V, +)$ and $(\mathbb{R}, \mathbb{R}^2, +)$ are vector spaces. – To verify if a given vector is a linear combination of other vectors – To differentiate linear dependent vectors from linear independent vectors – To determine the basis of a vector space and its dimension – To determine the image of vectors by using a linear transformation – To prove that $(\mathbb{R}, V, +)$ and $(\mathbb{R}, \mathbb{R}^2, +)$ are isomorphic vector spaces. 	<p>3. Algebra 3.1. Vector Space of real numbers</p> <ul style="list-style-type: none"> – Revision on vectors in Cartesian plane : addition of vectors and multiplication of vectors by a real number – The vector space $(\mathbb{R}, V, +)$ and the vector space $(\mathbb{R}, \mathbb{R}^2, +)$ – Linear combination of vectors – linear dependent vectors – linear independent vectors – generating set of vector space – basis of a vector space – dimension of a vector space – linear transformations: <ul style="list-style-type: none"> ▪ definition ▪ examples: enlargement, parallel projection, identical application, null application – isomorphism between the vector spaces $(\mathbb{R}, V, +)$ and $(\mathbb{R}, \mathbb{R}^2, +)$ 	<ul style="list-style-type: none"> – Emphasize the properties of basis of a vector space – Various exercises on linear dependent and independent vectors are required – Make a distinction among the different types of linear transformation <p>Establish a mapping from $(\mathbb{R}, V, +)$ to $(\mathbb{R}, \mathbb{R}^2, +)$ and prove that this mapping is linear and bijective.</p>
<ul style="list-style-type: none"> – To calculate the Scalar product of two vectors – To determine the magnitude of a vector 	<p>3.2. Euclidian vector space :</p> <ul style="list-style-type: none"> – Scalar products or dot product: <ul style="list-style-type: none"> ▪ definition, 	<ul style="list-style-type: none"> – Requirement of use of trigonometric concepts

<ul style="list-style-type: none"> - To establish the parametric and Cartesian equations of a straight line - To apply the scalar product to solve problems. - To establish the parametric and Cartesian equations of a circle 	<ul style="list-style-type: none"> ▪ vector notation ▪ properties : commutative, bilinear, positive ▪ modulus or magnitude (length) of a vector ▪ Expression of $\vec{u} \cdot \vec{v} = \ \vec{u}\ \cdot \ \vec{v}\ \cos t$, where t is the angle between \vec{u} and \vec{v} - Unit vector - Coordinates of a point - Lines in a plane: <ul style="list-style-type: none"> ▪ parametric and Cartesian equations of a straight line ▪ conditions for two lines to be parallel or perpendicular - Application of scalar product: <ul style="list-style-type: none"> ▪ relations in a triangle ▪ the median theorem ▪ distance between two points ▪ distance of a point from a straight line - Equations(parametric and Cartesian) of a circle 	<ul style="list-style-type: none"> - The teacher should prepare various exercises on <ul style="list-style-type: none"> ▪ relations in a triangle ▪ the median theorem ▪ distance between two points ▪ distance of a point from a straight line - The use of geometrical instruments by the teacher and by the learners is recommended - Prepare problems on determination of parametric and Cartesian equations - To visualize some concepts, the teacher should use drawings
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CHAPTER IV: TRIGONOMETRY

❖ **Duration: 35 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To use the trigonometric concepts and formulas in solving problem related to Trigonometry

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> - To express the angles in different units of measurements. 	<p>4. Trigonometry 4.1. Revision on Angle and its measurements: degree, grade, radians</p>	<ul style="list-style-type: none"> - Exercises leading to conversion of angle measurements
<ul style="list-style-type: none"> - To represent a given angle in a Cartesian plane - To locate the angle in a trigonometric circle. - To define sine, cosine, tangent and cotangent of any oriented angle. - To verify trigonometric identities using fundamental formula - To represent graphically the sine , cosine and tangent of an angle - To represent graphically the trigonometric functions by using coordinates - To apply trigonometric ratios to solve problems related to the right angled triangle and any other triangle. 	<p>4.2.The trigonometric circle</p> <ul style="list-style-type: none"> - The sine and cosine of an oriented angle in 1st, 2nd, 3rd, and 4th quadrants - The fundamental formula $\sin^2 x + \cos^2 x = 1$ and its applications - The tangent and cotangent of an oriented angle and its graphical interpretation - Definition of secant and cosecant of an oriented angle - Sine, cosine, tangent and cotangent of opposite, complementary and supplementary angles - Simple graphical representation of trigonometric functions by using coordinates 	<ul style="list-style-type: none"> - Use trigonometric circle to determine the sine, cosine, tangent and cotangent of an oriented angle - Using basic identities and fundamental trigonometric formulae in various exercises - Use of scientific calculator to calculate the sine, cosine, tangent and cotangent of a given angle - Plot graphs of trigonometric functions using coordinates. - The teacher should help learners to make emphasis on the sine, cosine, tangent and cotangent of principal angles (0°, 30°, 45°, 60°, 90°) - Various exercises on solving trigonometric equations and inequations using transformation formulae are required

<ul style="list-style-type: none"> – To use addition formulae, double angle (duplication) formulae and Simpson’s formulae to simplify the trigonometric expressions – To solve problems related to trigonometric equations and inequations. 	<p>4.3. Trigonometric ratios</p> <ul style="list-style-type: none"> – Trigonometric ratios in a right angled triangle – Trigonometric ratios in any other triangle – Solution of right angled triangle and any other triangle <p>4.4. Transformation formulae</p> <ul style="list-style-type: none"> – Addition formulae – Double angle (duplication) formulae – Simpson’s formulae <p>4.5. Trigonometric equations and inequations</p> <ul style="list-style-type: none"> – Basic equations and inequations – Equations and inequations reducing to basic equations and inequations 	
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CHAPTER V: DESCRIPTIVE STATISTICS

❖ **Duration: 14 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To determine and interpret the dispersion parameters of statistical series in one variable

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To calculate and interpret measures of central tendency of statistical data 	<p>5. Descriptive statistics</p> <p>5.1. Revision on measures of central Tendency : mean, mode, median and quartiles</p> <p>5.2. Measures of dispersion</p>	<ul style="list-style-type: none"> – Through various exercises involving data from real life, the teacher should lead learners to determine measures of: <ul style="list-style-type: none"> ▪ central tendency ▪ dispersion

<ul style="list-style-type: none"> – To find the measures of dispersion – To solve problems involving data representation and measures of dispersion 	<ul style="list-style-type: none"> – range, inter-quartiles range, – mean deviation, variance, standard deviation and Coefficient of Variation 	<ul style="list-style-type: none"> – The teacher should motivate learners to use scientific calculators while dealing with statistic data
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CHAPTER VI: MATRICES OF ORDER 2 AND ORDER 3

❖ **Duration: 28 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:
To carry out different operations on matrices of order 2 and order 3

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able:</p> <ul style="list-style-type: none"> – To determine the matrix of a linear transformation – To carry out operations on matrices – To calculate the determinants of matrices of order 2×2 and order 3×3 – To solve problems related to a system of 2 or 3 linear equations by using matrices – To calculate the inverse of a matrix of order 2×2 	<p>6. Matrices of order 2 and 3</p> <ul style="list-style-type: none"> – Matrices of a linear transformation <ul style="list-style-type: none"> • Definition of matrices (based on a linear transformation) – Operations on matrices: Addition, subtraction and multiplication of matrices – Determinants of matrices of order 2 and order 3 <ul style="list-style-type: none"> • Definition • Calculation and Properties of determinants – Solution of a system of 2 and 3 equations using Cramer's rule – Solution of simultaneous equations using matrices – inverses of matrices of order 2 	<ul style="list-style-type: none"> – Lead learners to form matrices based on a linear transformation – Various exercises on addition, subtraction and multiplication of matrices should be given to the learners – Exercises focusing on calculation of determinant and the system solving using inverse of matrices are required

PROPOSAL OF LESSONS DISTRIBUTION FOR SENIOR 4

First term

Weeks	Topics	Number of periods
1-2	Logic	14
3-4	Revision on sets of numbers and their structures	14
5-6	Polynomials	14
7-8	Set of real numbers	14
9-10	Equations, inequations and systems of the first degree	14
11-12	Revision and Exams	14
Total:12 weeks		84

Second term

Weeks	Topics	Number of periods
1-4	Quadratic equations, inequations and functions	28
5-9	Trigonometry	35
10	Algebra and plane geometry	7
11-12	Revision and Exams	14
Total:12 weeks		84

Third term

Weeks	Topics	Number of periods
1-4	Algebra and plane geometry	28
5-8	Matrices of order 2 and order 3	28
9-10	Statistics	14
11-12	Revision and Exams	14
Total: 12 weeks		84

PROGRAM FOR SENIOR 5

GENERAL OBJECTIVES BY THE END OF SENIOR FIVE

At the end of senior 5 the learner should be able to:

1. Study and to represent graphically a numerical function.
2. Apply the properties of real numbers (\mathbb{R}) in solving problems related to sequences and series.
3. Represent graphically and determine the linear adjustment of a given double series
4. Solve problems on combinations and permutations
5. Apply combinatorial concepts to solve problems related to probability
6. Carry out operations in real vector space \mathbb{R}^3 to find distances, amplitude of angles, areas and volumes and to establish algebraic equations of straight lines and planes

CHAPTER I: ANALYSIS OR CALCULUS

❖ **Duration: 112 Periods**

❖ **General objectives:** At the end of this chapter the learner should be able:

- To study and to represent graphically a numerical function.
- To apply the properties of real numbers (\mathbb{R}) in solving problems related to sequences and series

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit, student will be able :</p> <ul style="list-style-type: none"> - To determine if a given function is odd function, even function, periodic function - To find the period of a given function - To show if a given function decreases or increases on the interval - To determine the domain of different functions 	<p>1. Analysis 1.1.Numerical functions : Generalities:</p> <ul style="list-style-type: none"> – Neighborhood of a real number: Open intervals with centre a, Relationship between an open interval with centre a and absolute value : (point a not considered is $0 < x-a < \delta$ where $x \in I$ as part of the interval); – Definition of a numerical function: examples, parity of a numerical function: odd function, even function and periodic functions, increasing and decreasing functions, domain of definition of a function. 	<ul style="list-style-type: none"> – The teacher should make revision on intervals before introducing neighborhood of a real number – The teacher should prepare various exercises to help learners to master the use of compound concepts in this unit such as: domain of a function, odd function, even function , periodic function, inverse and composition of functions and monotonic function

<ul style="list-style-type: none"> - To evaluate correctly the given limits 	<p>1.2 Limits :</p> <ul style="list-style-type: none"> - Concept of a limit with examples, - Definition of the limit based on the concept of neighborhood, - Defining limits by use of “ε and δ” (epsilon and delta) - Right hand limits and left hand limits - Properties of finite limits, uniqueness of a limit, limit of a constant function, limit of an identity function, conservation of an inequality, sandwich theorem, - Operations on limits - Extension of the concept of limit: when values of the variable or those of the function tend towards infinity, - Indeterminate cases : $\frac{\infty}{\infty}$; $\frac{0}{0}$; $\infty - \infty$; $0 \cdot \infty$ 	<ul style="list-style-type: none"> - Before introducing the notation $\lim_{x \rightarrow a} f(x) = b$ the teacher should give examples showing that if x takes values very closed to a then $f(x)$ takes values which are very closed to b - It is advisable to develop the definition of limits using ε and δ by illustrating the idea with some simple examples and exercises based on neighborhood - The teacher should prepare examples and exercises to help learners to use correctly the definition of limit - The teacher should ensure that the limits to be evaluated in exercises help learners while dealing with indicated indeterminate forms.
<ul style="list-style-type: none"> - To study the continuity of a given function at a given point or a given interval 	<p>1.3 Continuity :</p> <ul style="list-style-type: none"> - Continuity of a function at a point, discontinuity at a point - Continuity of a function on a subset of IR - Properties of continuous functions (proofs not required) - Operations on continuous functions - Theorem of intermediate values - Monotonous functions - Inverse of a function which is continuous and strictly monotonous by increasing or decreasing - Bounded functions: upper bound and lower bound 	<ul style="list-style-type: none"> - The teacher should help learners to illustrate graphically the discontinuity points of a curve - It is better to give various problems on the study of continuity of different functions

	<ul style="list-style-type: none"> – Maximum and minimum of a continuous function over an open interval $] a, b [$: properties (proofs not required) 	
<ul style="list-style-type: none"> – To find the eventual asymptotes of a given function 	<p>1.4. Asymptotes:</p> <ul style="list-style-type: none"> – Definition: asymptotes on a curve, types of asymptotes, – Determination of vertical, horizontal and oblique asymptotes, 	<ul style="list-style-type: none"> – The teacher should lead learners to illustrate graphically when a straight line is asymptote to a curve – The use of limits is required to determine vertical, horizontal and oblique asymptotes through various exercises
<ul style="list-style-type: none"> – To find the first derivative of a given function using definition – To determine derivatives using formulae – To determine the equation of tangent and normal at a point on a curve. – To use verify correctly Rolle's theorem and Lagrange's theorem – To use correctly Hospital's rule for calculating limits. 	<p>1.5 .Differentiation :</p> <ul style="list-style-type: none"> – Derivative of a function at a point, – Geometric interpretation of a derivative of a function at a point, – Kinematical meaning of a derivative: velocity at a time t, – Differentiability and continuity of a function, – Derivative of a function, – Theorems on differentiable functions: derivative of a constant, of a sum, of a product, and a quotient of functions, – Differentiation of some important functions: power function, inverse function, composite function, circular functions, parametric functions and implicit functions. – Successive differentiation, – Application of derivative: <ul style="list-style-type: none"> – Tangent and normal at a point of a function – Rolle's theorem, Lagrange's mean value theorem and Hospital's theorem (proof not required), – Increasing and decreasing functions, 	<ul style="list-style-type: none"> – The teacher should lead learners to identify geometrical interpretation of derivative of a function at a point – While denoting derivatives, the teacher should help learners to be familiar with the notations y' and $\frac{dy}{dx}$ – The teacher should prepare and give various exercises on differentiation of functions in different forms – The teacher should give problems with data from real life as application of derivatives emphasizing on the concept of maximum and minimum – The teacher should give various exercises concerning the use of Hospital's rule as application of

<ul style="list-style-type: none"> – To determine whether a turning point of a given function is a maximum or a minimum point. – To solve problems involving maximum or minimum values – To study the concavity of a given function and find its inflection points. 	<ul style="list-style-type: none"> – Minimum and maximum points of a function, – Concavity, inflection point on a curve by use of the second derivative, 	<p>derivatives</p>
<ul style="list-style-type: none"> – To study different functions and sketch their curves. 	<p>1.6. Study of a function and curve sketching</p> <ul style="list-style-type: none"> – Types of functions to consider : <ul style="list-style-type: none"> ▪ Polynomial functions ▪ Rational functions $x \rightarrow \frac{ax+b}{cx+d}, \quad (a \neq 0, c \neq 0)$ $x \rightarrow \frac{ax^2+bx+c}{dx+e} \quad (a \neq 0, d \neq 0)$ $x \rightarrow \frac{ax^2+bx+c}{dx^2+ex+f}, \quad (a \neq 0, d \neq 0)$ ▪ Irrational functions : functions in square root form and cube root form ▪ Functions including one or several absolute values. ▪ Circular functions, ▪ Inverse trigonometric functions. 	<ul style="list-style-type: none"> – Ensure that each required form of function is sketched through various exercises
<ul style="list-style-type: none"> – To use the mathematical induction in demonstrations – To determine if a given sequence increases or decreases, converges or not – To identify characteristics of Arithmetic progressions.. 	<p>1.7. Sequences :</p> <ul style="list-style-type: none"> – Mathematical induction – Numerical sequences – General concepts: definitions, increasing sequences, decreasing sequences, operations and order on sequences, convergence of sequences, limits of sequences, comparison of 2 sequences – Arithmetic sequences : definition, sense of variation, 	<ul style="list-style-type: none"> – After defining sequences the teacher should lead learners to give their own examples of sequences – The teacher should emphasize on

CHAPTER II: DESCRIPTIVE STATISTICS

❖ **Duration: 21 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:
To represent graphically and to determine the linear adjustment of a given double series

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able:</p> <ul style="list-style-type: none"> - To draw the scatter diagram of given statistical series in two quantitative variables - To determine the linear regression line of a given double series. - To calculate a linear correlation coefficient of a given double series 	<p>2. Descriptive statistics</p> <p>2.1. Double series</p> <ul style="list-style-type: none"> - Statistical series in two quantitative variables: scatter diagram. - Linear adjustment: least squares' method, linear correlation and regression line 	<ul style="list-style-type: none"> - The teacher should guide learners to suggest different situations involving statistical series in two quantitative variables - It is advisable to use data from real life while preparing exercises and problems - The use of ICT tools is suggested

CHAPTER III: COMBINATORIAL AND PROBABILITY

❖ **Duration: 35 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

- To solve problems on combinations and permutations
- To apply combinatorial concepts to solve problems related to probability

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> - To solve problems involving factorial notation - To determine the number of permutations of n different objects taken r at a time. - To determine the number - To determine the number of permutations 	<p>3. Combinatorial and probability</p> <p>3.1. Permutations and combinations</p> <ul style="list-style-type: none"> - Factorial notation - Calculation of the number of permutations (arrangements) of n elements taken r at a time. - Calculation of the number of permutations of n elements taken n at a time ($n!$) - Simple combinations, 	<ul style="list-style-type: none"> - The teacher should help learners to be familiar with factorial notation and calculations through different exercises - By use of examples from real life, the teacher should introduce the concepts of permutations and combinations. It is also advisable

<p>of n different objects</p> <ul style="list-style-type: none"> – To determine the number of permutations of n different objects taken r at a time for given conditions – To determine the number of combinations of r objects chosen from n different objects. – To determine the number of combinations r objects chosen from n different objects for given conditions – To use properties of combinations for finding coefficients in Pascal's triangle. – To solve problems involving permutations and combinations. 	<ul style="list-style-type: none"> – Calculations of the number of combinations of n elements taken r at a time. – Properties of combinations – Pascal's triangle 	<p>to go out of classroom for visualizing permutation of n elements through different activities</p> <ul style="list-style-type: none"> – The teacher prepares various exercises and problems on permutations and combinations – The teacher should help learners to construct themselves Pascal's triangle by use of properties of combinations
<ul style="list-style-type: none"> – To apply the binomial formula in different exercise 	<p>3.2. Binomial theorem</p> <ul style="list-style-type: none"> – Binomial theorem : general notation by use of the summation sign (Σ) – Properties of binomial coefficients 	<ul style="list-style-type: none"> – The teacher should prepare various exercises on application of binomial theorem
<ul style="list-style-type: none"> – To determine the sample space of an experiment – To explain different concepts relating to events – To find the probability of different events – To determine the probability of an event with an equiprobable sample space. – To solve different exercises on conditional probability and independent events – To use correctly Bayes's theorem in solving problems 	<p>3.3. Probability</p> <ul style="list-style-type: none"> – Introduction – Random experiment – Sample space – Events: definition, particular cases of events – Probability of events: introduction, definition, properties – Equiprobability – Conditional probability : definition, Independent events and Bayes's theorem 	<ul style="list-style-type: none"> – The teacher should lead learners to deal with random experiments – Through various examples and exercises, particular events and their properties are introduced – Various exercises on calculation of probability of events are required – The teacher should help learners to solve problems involving simultaneous draws, successive draws (with or without replacing), conditional probability, dependent events and to use correctly Bayes's theorem

CHAPTER IV: SPACE GEOMETRY

❖ **Duration: 42 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To carry out operations in real vector space \mathbb{R}^3 to find distances, amplitude of angles, areas and volumes and to establish algebraic equations of straight lines and planes

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able:</p> <ul style="list-style-type: none"> - To construct straight lines and planes in space according to various relative positions - To construct images of figures under parallel projection in space 	<p>4.Space geometry</p> <p>4.1.The space E</p> <ul style="list-style-type: none"> - Axioms: Relative positions of lines and planes. - Parallel and Orthogonal projections in space 	<ul style="list-style-type: none"> - The teacher should help learners to identify relative positions of lines and planes by drawings - The teacher should help learners to construct images of figures by parallel and orthogonal projections
<ul style="list-style-type: none"> - To verify if \mathbb{R}^3 has a vector space structure 	<p>4.2. The vector space E_0.</p> <ul style="list-style-type: none"> - The vector space $(\mathbb{R}, E, +)$ - Calculations in the vector space $(\mathbb{R}, E_0, +)$ - The vector space $(\mathbb{R}, \mathbb{R}^3, +)$ or the real vector space \mathbb{R}^3 - Isomorphism between E_0 and \mathbb{R}^3 	<ul style="list-style-type: none"> - Make revision on abelian group and field - Prepare various exercises on vector space \mathbb{R}^3
<ul style="list-style-type: none"> - To establish equations of lines and of planes in space. 	<p>4.3.Lines and planes in space</p> <ul style="list-style-type: none"> - Equations of lines in space - Equations of planes in space 	<ul style="list-style-type: none"> - Help learners to determine vectorial , parametric and Cartesian equations of line and plane through various exercises
<ul style="list-style-type: none"> - To calculate scalar product of vectors in space - To verify orthogonality by use of scalar product 	<p>4.4.The scalar product</p> <ul style="list-style-type: none"> - The scalar product in space E_0 - Properties of the scalar product - Norm or magnitude of a vector. - Cosine of a pair of vectors - Orthogonality of vectors 	<ul style="list-style-type: none"> - Prepare and give various exercises on scalar product and its application

	<ul style="list-style-type: none"> – Orthonormal basis in space E_0 – Scalar product in \mathbb{R}^3 	
<ul style="list-style-type: none"> – To verify the orthogonality of two lines, of a line and a plane and of two planes. – To determine the angle between two lines and between two planes. 	<p>4.5. Orthogonality</p> <ul style="list-style-type: none"> – Orthogonal lines – Perpendicular lines to a plane – Perpendicular planes – Parallel planes – Distance of a point from a plane – Angles between lines and angles between planes. 	<ul style="list-style-type: none"> – By use of drawings, local materials, illustrate the concepts to be studied in this unit – Various exercises are required
<ul style="list-style-type: none"> – To determine the intersection of two planes – To determine the intersection of a line and a plane 	<p>4.6. Intersection of planes</p> <ul style="list-style-type: none"> – Intersection of two planes – Intersection of a line and a plane 	<ul style="list-style-type: none"> – By use of drawings, local materials, and simple examples illustrate the concepts to be studied in this unit – Various exercises involving equations of lines and planes to determine intersection
<ul style="list-style-type: none"> – To identify the characteristics of a sphere – To use correctly the equation of sphere – To determine the intersection of a sphere and a plane – To determine the intersection of a sphere and a line. 	<p>4.7. Sphere</p> <ul style="list-style-type: none"> – Definition and equation of a sphere – Intersection of a sphere and a plane – Intersection of a sphere and a line 	<ul style="list-style-type: none"> – Establish the equation of sphere, by use of the concept of distance in space – Various exercises about sphere
<ul style="list-style-type: none"> – To determine the cross product of two vectors in space – To determine the mixed product of vectors in space. 	<p>4.8. The vector product or cross product</p> <ul style="list-style-type: none"> – Vector product in space – Mixed product of vectors in space. 	<ul style="list-style-type: none"> – The teacher should help learners to use the right hand rule to illustrate the direction of cross product – Various exercises on vector product in space and mixed product of vectors and their applications are required

PROPOSAL OF LESSONS DISTRIBUTION FOR SENIOR 5

First term

Weeks	Topics	Number of periods
1-2	Generalities on numerical functions	14
3-6	Limits and continuity	28
7	Asymptotes	7
8-10	Differentiation	21
11-12	Revision and Exams	14
Total: 12 weeks		84

Second term

Weeks	Topics	Number of periods
1-4	Study of function and curve sketching	28
5-6	Sequences	14
7-10	Space geometry	28
11-12	Revision and Exams	14
Total: 12 weeks		84

Third term

Weeks	Topics	Number of periods
1-2	Space geometry	14
3-5	Descriptive statistics	21
6-10	Combinatorial and probability	35
11-12	Revision and Exams	14
Total: 12 weeks		84

PROGRAM FOR SENIOR 6

GENERAL OBJECTIVES BY THE END OF SENIOR SIX

At the end of senior 6 the learner should be able:

1. Utilize the algebraic, trigonometric and exponential forms of a non-zero complex number to solve problems in trigonometry, geometry and in the factorization of polynomials;
2. Utilize the definitions and properties of vector space and linear functions to solve problems given in vector space \mathbb{R}^n ;
3. Study and represent logarithmic and exponential functions and to apply them in other scientific domains;
4. Apply Taylor's formula in establishing limited development of the functions with real variables;
5. Calculate the integrals of functions and apply them in various domains;
6. Solve simple differential equations of 1st and 2nd order with constant coefficients;
7. Determine and establish equations of a conic;
8. Calculate and interpret the parameters of a random variable.

CHAPTER I: COMPLEX NUMBERS

❖ **Duration: 28 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To utilize the algebraic, trigonometric and exponential forms of a non-zero complex number to solve problems in trigonometry, geometry and in the factorization of polynomials

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To define a complex number in algebraic form – To calculate the sum / difference and the product of complex numbers 	<p>1. Complex numbers</p> <p>1.1. The commutative field of complex numbers</p> <ul style="list-style-type: none"> – Definition of a complex number – Notation of the set of complex numbers – Addition in \mathbb{C} and its properties – Multiplication in \mathbb{C}, and its properties – Commutative field $(\mathbb{C}, +, \cdot)$ – Isomorphism between \mathbb{R} and $\mathbb{C}_1 = \{(a, 0); a \in \mathbb{R}\}$ 	<ul style="list-style-type: none"> – Before introducing complex numbers, the teacher should make revision on properties of addition and multiplication in \square^2 – The teacher should also make revision on properties of linear application before introducing the isomorphism between \mathbb{R} and

<ul style="list-style-type: none"> - To determine the real and imaginary parts of a complex number - To determine the conjugate and modulus of complex numbers - To apply properties of conjugate and modulus of complex numbers in solving simple equations - To represent a complex number in Argand diagram - To calculate a square root of a complex number - To solve quadratic equations in \mathbb{C} - To define a complex number in trigonometric form - To determine a modulus and an argument of product, quotient of two complex numbers in trigonometric form - To determine a modulus and an argument of power of complex number - To convert a complex number from a trigonometric form to an algebraic form and vice versa - To define a complex number in Exponential form 	<ul style="list-style-type: none"> - Algebraic form of a complex number : <ul style="list-style-type: none"> • real and imaginary parts • conjugate and modulus of complex numbers, notation and properties 1.2. Geometric representation of a complex number: <ul style="list-style-type: none"> - Location of points and vectors in a complex plane or Argand diagram. 1.3. Calculations in the field of complex numbers <ul style="list-style-type: none"> - Square root of a complex number - Quadratic equations in \mathbb{C} 1.4. Trigonometric forms of complex numbers <ul style="list-style-type: none"> - Modulus and Argument of a complex number - Trigonometric form of product, and quotient of two complex numbers - Power of complex number in trigonometric form - DeMoivre's theorem. - Changing a complex number from a trigonometric form to an algebraic form and vice versa - Exponential form of a complex number 	<p>$\mathbb{C}_1 = \{(a, 0); a \in \mathbb{R}\}$</p> <ul style="list-style-type: none"> - During introduction, the teacher should help learners to identify the insufficiency of \mathbb{R} set while calculating the square root of negative real numbers - The teacher should prepare various exercises to help learners to master calculations in \mathbb{C} - The teacher should help learners to relate the representation of a point in Cartesian plane to the representation of complex number in complex plane or Argand diagram - While calculating the square root of $z = a + bi$, it is advisable to solve the system $\begin{cases} x^2 - y^2 = a \\ x^2 + y^2 = \sqrt{a^2 + b^2} \\ 2xy = b \end{cases}$ <p>Where $x + yi$ is the square root of z. Make sure that the choice of x and y depends on the sign of b</p> - The teacher should help learners to identify that considering the sign of real part and imaginary part will facilitate to determine the position of argument of a complex number in Argand diagram
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<ul style="list-style-type: none"> - To determine the n^{th} roots of a complex number - To represent graphically the n^{th} roots of a unit number - To apply DeMoivre's formula in calculating sine, cosine, tangent and cotangent of multiple angles of a given angle - To solve trigonometric equations and inequations in \mathbb{C} - To Construct regular polygons with n sides - To determine the length of one side of a polygon and the length of its apothem in terms of the radius of the circumscribed circle. - To factorize polynomials in \mathbb{C} 	<p>1.5. The n^{th} root of a complex number</p> <ul style="list-style-type: none"> - The n^{th} roots of a unit number - Graphical representation - The n^{th} roots of a complex number. <p>1.6. Application of complex numbers</p> <ul style="list-style-type: none"> - Calculation of sine, cosine, tangent, cotangent of multiple angles of a given angle and demonstrating the trigonometric identities by use of DeMoivre's theorem - Linearization of trigonometric polynomials - Solution of simple trigonometrical equations and inequations - Construction of regular polygons with n sides and determination of the length of the sides and the length of apothem in terms of the radius of the circumscribed circle. - Factorisation of polynomials in \mathbb{C} 	<ul style="list-style-type: none"> - Prepare various exercises relative to the changing quotient, product, power of complex numbers in different forms - The teacher should help learners to identify that the n^{th} roots of unit are the vertices of a regular polygon inscribed in a circle with radius $r = 1$ - The teacher should prepare various exercises related to the application of complex numbers - While solving the simple trigonometric equations, make sure that $a \cos x + b \sin x = c$ is also solved
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CHAPTER II: LINEAR ALGEBRA

❖ **Duration: 28 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To utilize the definitions and properties of vector space and linear functions to solve problems given in vector space \mathbb{R}^n

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To define the vector spaces \mathbb{R}^2 and \mathbb{R}^3 – To carry out operations on linear combinations – To determine linear dependent and linear independent vectors – To determine Generating set of vectors – To determine Basis and dimension of a vector space <ul style="list-style-type: none"> – To prove that the sum of 2 sub vector spaces is also a vector space <ul style="list-style-type: none"> – To define a linear mapping – To apply the properties of a linear mapping – To determine the matrix of a linear mapping – To determine the kernel and image of a linear mapping – 	<p>2. Linear algebra</p> <p>2.1. Structure of the vector space \mathbb{R}^n</p> <ul style="list-style-type: none"> – Vector spaces \mathbb{R}^2 and \mathbb{R}^3 – Linear combination – Linear dependence – Linear independence – Generating set of vectors – Basis and dimension of a vector space. <p>2.2. Sub-vector space</p> <ul style="list-style-type: none"> – Definition – Sum of 2 sub vector spaces theorem (Grassman theorem) – Supplementary vector spaces <p>2.3.Linear mappings</p> <ul style="list-style-type: none"> – Definition – Properties – Matrix of a linear mapping – Kernel and image of a linear mapping 	<ul style="list-style-type: none"> – The teacher should lead learners to use determinant to verify linear dependency and linear independency of vectors <ul style="list-style-type: none"> – Help learners to realize that a sub vector space is also a vector space – Lead learners to verify that a non empty subset of a given vector space is its subspace by use of linearity property through different exercises <ul style="list-style-type: none"> – The teacher should focus on identity, null application, and enlargement. – Help learners to realize that if $f : E \rightarrow F$: Linear mapping then , <ul style="list-style-type: none"> ▪ $\ker f$ is a subspace of E ▪ $\text{Im} f$ is a subspace of F

<ul style="list-style-type: none"> - To calculate a determinant of a matrix of order $n \times p$ - To solve a system of n linear equations in p unknowns 	<ul style="list-style-type: none"> - Matrices and determinants: <ul style="list-style-type: none"> ▪ Revision ▪ Generalization - System of n linear equations in p unknowns 	<ul style="list-style-type: none"> - Various exercises on determination of kernel, image of a linear transformation and their dimensions are required - While solving systems of linear equations, it is advisable to use Gauss method and Cramer's method where possible
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CHAPTER III: CALCULUS OR ANALYSIS

❖ **Duration: 98 Periods**

❖ **General objectives:** At the end of this chapter the learner should be able:

- To study and represent logarithmic and exponential functions and to apply them in other scientific domains
- To apply Taylor's formula in establishing limited development of the functions with real variables
- To calculate the integrals of functions and apply them in various domains

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> - To define Neperian logarithm function - To represent graphically $y = \ln x$ - To solve logarithmic equations and inequations by use of properties - To calculate the derivative of logarithmic functions in base e - To define an exponential function in base e - To study and represent graphically $y = e^x$ 	<p>3. Calculus(Analysis)</p> <p>3.1.Logarithmic and exponential functions</p> <ul style="list-style-type: none"> - Neperian logarithm function: definition, properties, the number e, study and graphical representation of $y = \ln x$, logarithmic equation and inequations, differentiation of $\ln u$ on an interval I, where u is differentiable on I, study of logarithmic functions in base e - Exponential functions in base e: definition, notation, properties, study of the function $y = e^x$, exponential equations and inequations, differentiation of e^u where u is differentiable on I 	<ul style="list-style-type: none"> - The teacher should help learners to find properties of logarithm to any base through properties of logarithm to base 10 - The teacher should lead learners to establish the relation between logarithm to base a and logarithm to base e where $a > 0, a \neq 1$ - On the graphical

<ul style="list-style-type: none"> - To solve exponential equations and inequations by use of properties - To calculate the derivative of exponential functions in base e - To define a logarithmic function in base a - To study and represent graphically $y = \log_a^x$ - To solve logarithmic equations and inequations by use of properties - To calculate the derivative of logarithmic functions in base a - To define an exponential function in base a - To study and represent graphically $y = a^x$ - To solve exponential equations and inequations in base a by use of properties - To calculate the derivative of exponential functions in base a - To establish the relationship between exponential functions $y = a^x$ and $y = e^x$ - To study and represent graphically logarithmic and exponential functions in base a - To calculate limits of indeterminate forms: $1^\infty, 0^0, \infty^0, \infty^\infty$ 	<ul style="list-style-type: none"> - Logarithmic functions in base a (where $a > 0, a \neq 1$): definition, properties, study of the function $y = \log_a x$, change of base, logarithmic equations and inequations in base a, differentiation of $\log_a u$ where u is differentiable on I - Exponential functions in base a: definition, notation, properties, study of $y = a^x$, exponential equations and inequations, differentiation of a^u where u is differentiable on I - Relationship between exponential functions in base a, and in base e, - Study of logarithmic and exponential functions in base a - Limits of indeterminate forms: $1^\infty, 0^0, \infty^0, \infty^\infty$ 	<p>representation of $y = \ln x$, the illustration of</p> $\lim_{x \rightarrow 0^+} \ln x = -\infty,$ $\lim_{x \rightarrow +\infty} \ln x = +\infty, \ln x < 0 \text{ for } 0 < x < 1 \text{ and } \ln x > 0 \text{ for } x > 1$ <p>is suggested</p> <ul style="list-style-type: none"> - The teacher should lead learners to evaluate limits, to differentiate, to solve equations and inequations involving logarithmic and exponential expressions through various exercises; function study of logarithmic and exponential functions is also required - While dealing with limits of indeterminate forms: $1^\infty, 0^0, \infty^0, \infty^\infty$, the teacher should prepare various exercises for each case. The use of Napierian logarithm and Hospital's rule is required. Particularly, the following limits are to be considered: $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$ and $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$
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<ul style="list-style-type: none"> - To apply Taylor's formula in limited development of functions - To calculate the approximated numerical values of functions in a given point - To define a differential of function - To calculate the differential of various functions - To define a primitive function - To apply immediate primitives in exercises - To calculate integrals by decomposition method - To calculate integrals by substitution method 	<p>3.2. Taylor and Mac-Laurin polynomials</p> <ul style="list-style-type: none"> - Taylor's formula (proof not required) - Application of Taylor's formula - Taylor's polynomial of $\sin x, \cos x, e^x, \ln(1+x)$ in the neighborhood of $x=0$ - Calculation of sine, cosine of an angle (proof required) - Taylor's polynomial for given functions, numerical values of functions. - Limited Development of functions. <p>3.3. Differential calculus</p> <ul style="list-style-type: none"> - Definition, differential of a sum, product, quotient of differentiable functions. - Differential of a function in square root form - Differential of functions with rational powers - Differential of composite functions - Differential of trigonometric functions - Differential of inverse trigonometric functions - Differential of exponential functions - Differential of logarithmic functions - Application of differentials <p>3.4. Primitive Functions</p> <ul style="list-style-type: none"> - Definition - Properties: continuity of primitive of a function, set of primitive functions, immediate primitives - Techniques of integration: <ul style="list-style-type: none"> ▪ integration by decomposition: integration of a sum of functions, integration of a product of functions by a real number ▪ integration by change of variable, ▪ integration by parts 	<ul style="list-style-type: none"> - Various exercises on application of Taylor's and Mac-Laurin's formulae are required - Teacher should make revision on derivatives before introducing the concept of differential of functions - Prepare and give various exercises on differential of different types of functions - The teacher should lead learners to calculate indefinite integrals by decomposition, by change of variable and by parts through various exercises
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<ul style="list-style-type: none"> - To calculate integrals by parts - To calculate integrals of rational functions - To calculate integrals of irrational functions - To calculate integrals of trigonometric functions - To define a definite integral - To calculate definite integrals by applying their properties - To calculate the area of a plane surface by use of definite integrals - To calculate the volume of a solid of revolution by use of definite integrals - To calculate the length of a curved 	<ul style="list-style-type: none"> - Some special integrals: <ul style="list-style-type: none"> ▪ Rational functions: the general rule (proof not required) based on the decomposition of proper fraction into a sum of simple fractions, ▪ Integration of functions given in improper fractions, ▪ Integration of irrational functions: $f(x) = \sqrt{ax+b},$ $f(x) = \sqrt{\frac{ax+b}{cx+d}},$ $f(x) = \sqrt{a^2-x^2}$ ▪ Integration of trigonometric functions: odd functions in $\cos x$, odd functions in $\sin x$. 3.5. Definite integrals <ul style="list-style-type: none"> - Definition - Properties: <ul style="list-style-type: none"> ▪ Linearity, ▪ Permutation of bounds ▪ Particular case where bounds are equal, ▪ Additivity of integration on intervals (Chasles relation), ▪ Positivity , ▪ Mean value theorem - Methods of integration, - Applications definite integrals : <ul style="list-style-type: none"> ▪ Calculation of area of a plane surface ▪ Calculation of volume of a solid of revolution ▪ Calculation of the length of curved surface 	<ul style="list-style-type: none"> - While integrating, the teacher should lead learners to change variable as follow: $\sqrt{ax+b}, \quad t^2 = ax+b$ $\sqrt{\frac{ax+b}{cx+d}}, \quad t^2 = \frac{ax+b}{cx+d}$ $\sqrt{a^2-x^2}, \quad x = a \sin t$ - While integrating, odd functions in $\cos x$ we change variable by using $t = \sin x$, for odd functions in $\sin x$ we change variable by using $t = \cos x$. - The teacher should lead learners to calculate definite integrals by decomposition , by change of variable and by parts through various exercises - While calculating the area,
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surface by use of definite integrals		<p>volume and the length, the teacher should also lead learners to:</p> <ul style="list-style-type: none"> ▪ determine the area of square, rectangle, trapezium, triangle, and circle ▪ determine the volume of sphere, cylinder and cone ▪ determine the circumference of a circle
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CHAPTER IV: DIFFERENTIAL EQUATIONS

❖ **Duration: 21 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To solve simple differential equations of 1st and 2nd order with constant coefficients

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To define a differential equation of the 1st order – To define a differential equation of the 2nd order – To solve differential equations with separable variables – To solve simple homogeneous differential equations – To solve linear differential equations of 	<p>4. Differential equations</p> <p>4.1. Introduction</p> <ul style="list-style-type: none"> – Definition and terminology – Examples of differential equations of the 1st and second order <p>4.2. Differential equations of the 1st order</p> <ul style="list-style-type: none"> – Differential equations with separable variables – Simple homogeneous differential equations – Linear differential equations of the 1st order. 	<ul style="list-style-type: none"> – The teacher should help learners to distinguish the 1st and 2nd order of differential equations through various examples – The teacher should prepare exercises on verifying whether a given function is a solution of a given differential equation – In solving linear differential

<p>the 1st order</p> <ul style="list-style-type: none"> – To solve linear differential equations of the 2nd order with constant coefficient 	<p>4.3. Differential equations of the 2nd order</p> <ul style="list-style-type: none"> – Linear equations with constant coefficients: <ul style="list-style-type: none"> ▪ The right hand side is equal to zero ▪ The right hand side is a polynomial function ▪ The right hand side is a trigonometric function ▪ The right hand side is an exponential function 	<p>equations of the 1st order it is advisable to find out the integrating factor leading to differential equations with separable variables</p> <ul style="list-style-type: none"> – The teacher should prepare various exercises on linear differential equations of the 2nd order with constant coefficients to be studied
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CHAPTER V: CONICS

❖ **Duration: 21 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:

To determine and establish equations of a conic

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To define a parabola – To determine the Cartesian and parametric equations of a parabola – To find out the tangents, axis of symmetry of a parabola – To define an ellipse – To determine the Cartesian and parametric equations of an ellipse – To find out the tangents, axis and centre of symmetry of an ellipse 	<p>5. Conics</p> <p>5.1. The parabola:</p> <ul style="list-style-type: none"> – Definition: focus and directrix of a parabola – Cartesian and parametric equations – Tangents, axis of symmetry <p>5.2. The ellipse</p> <ul style="list-style-type: none"> – Definition: foci and directrices of an ellipse – Cartesian and parametric equations – Tangents, axes and centre of symmetry. <p>5.3. The hyperbola</p>	<ul style="list-style-type: none"> – While defining analytically each conic it's better to talk about vertices, foci, directrices and axes

<ul style="list-style-type: none"> – To define an hyperbola – To determine the Cartesian and parametric equations of an hyperbola – To find out the asymptotes, tangents, axes and centre of symmetry of an hyperbola – To determine an eccentricity of a conic – To determine the nature of a conic based on its eccentricity – To define and represent graphically a point in polar coordinates – To convert polar coordinates in Cartesian coordinates and vice versa – To determine the equations of straight line, circle and conics in polar coordinates 	<ul style="list-style-type: none"> – Definition: foci and directrices of an hyperbola – Cartesian and parametric equations – Asymptotes, tangents, axes and centre of symmetry 5.4. Conics and eccentricity – Eccentricity – Nature of a conic 5.5. Conics and polar coordinates – Definition of polar coordinates – Conversion of polar coordinates in Cartesian coordinates and vice versa – Straight line, circle and conics in polar coordinates 	<ul style="list-style-type: none"> – The teacher should help learners to determine the type of a given conic considering the value of eccentricity e: <ul style="list-style-type: none"> ▪ Ellipse : $0 < e < 1$ ▪ Hyperbola: $e > 1$ ▪ Parabola: $e = 1$ ▪ Circle : $e = 0$ – The teacher should prepare and give various exercises on conversion of polar coordinates in cartesian coordinates and vice versa
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CHAPTER VI: PROBABILITY

❖ **Duration: 14 Periods**

❖ **General objective:** At the end of this chapter the learner should be able:
To calculate and interpret the parameters of a random variable

Specific objectives	Contents	Suggested teaching and learning activities
<p>At the end of the unit the learner will be able :</p> <ul style="list-style-type: none"> – To establish the table of distribution of probability – To calculate Mathematical expectation, variance, standard deviation of a random variable – To apply Bernoulli’s law in solving various problems – To apply Poisson’s law in solving various problems – To determine a function of density of probability of a continuous random variable – To prove if a given function is a density of probability on a given interval – To calculate mathematical expectation, variance, standard deviation of a continuous random variable 	<p>6. Probability</p> <p>6.1. Discrete and finite Random variables</p> <ul style="list-style-type: none"> – Distribution of probability – Mathematical expectation, variance, standard deviation – The Law of Bernoulli: Binomial distribution and its properties – Uncountable infinite discrete case: law of Poisson <p>6.2. Continuous Random variables :</p> <ul style="list-style-type: none"> – Continuous random variable – Functions of density of probability – Mathematical expectation, variance and standard deviation. 	<ul style="list-style-type: none"> – While calculating mathematical expectation, variance and standard deviation, the teacher should help learners to be familiar with the use of the table of distribution of probability through various exercises – The teacher should lead learners to represent the distribution of probability by use of bar diagram through exercises – In applying Bernoulli’s law, the teacher should give problems involving success or failure.

PROPOSAL OF LESSONS DISTRIBUTION FOR SENIOR 6

First term

Weeks	Topics	Number of periods
1-4	Complex numbers	28
5-8	Logarithms and exponential functions	28
9	Taylor and Mac-Laurin series	7
10	Differential calculus and primitive functions	7
11-12	Revision and Exams	14
Total: 12 weeks		84

Second term

Weeks	Topics	Number of periods
1-4	Primitive functions	28
5-8	Definite integrals	28
9-10	Differential equations	14
11-12	Revision and Exams	14
Total: 12 weeks		84

Third term

Weeks	Topics	Number of periods
1	Differential equations	7
2-5	Linear algebra	28
6-8	Conics	21
9-10	Probability	14
11-12	Revision and Exams	14
Total: 12 weeks		84

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